

Octet and Decuplet Baryons in the Hadronized Nambu–Jona-Lasinio Model with Proper Spin Projection [†]

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Abstract

Octet and Decuplet baryons are described within the hadronized NJL model as diquark–quark states, which are bound by quark exchange. Including scalar and axial–vector diquark correlations, we project the previously obtained relativistic Faddeev equation in a Poincaré invariant fashion onto good spin, using a static approximation to the exchanged quark. The resulting equations for the spin $1/2$ octet and the spin $3/2$ decuplet are solved numerically.

1. Introduction

Except for some lattice calculations at low energies, a description of hadrons within QCD is still not feasible. One therefore resorts to effective quark models, which mimic the low energy flavor dynamics of QCD. In this respect the Nambu–Jona-Lasinio (NJL) model [1] has proved very successfully in the past. The success of this model is mainly due to its chiral symmetry, which almost entirely determines the low energy meson dynamics. The color–NJL model (see equation (1) below) describes quarks interacting via a local color–octet current interaction, which for a large number of colors N_C reduces to an attractive quark–antiquark color–singlet interaction giving rise to the formation of mesons. The bosonization of this model leads to an effective meson theory [2], which is in fair agreement with the experimental findings. Within this effective meson theory baryons appear as solitons of the meson fields [3], consistent with Witten’s conjecture on large N_C –QCD [4]. For a finite number of colors the NJL model contains not only attractive quark–antiquark correlations but also attractive quark–quark correlation. In this case the model can be hadronized i.e. converted into an effective hadron theory, which contains beside mesons also explicit baryon fields [5]. The latter appear as diquark–quark states, which are bound by quark exchange ¹. Similar investigations have been also performed in the so–called global color model [8], and in a NJL model with three body forces [9]. All these approaches give rise to a relativistic Faddeev equation for baryons.

The relativistic Faddeev equation of the NJL model has been solved for spin $1/2$ baryons in [10] including only scalar diquarks (which are the counterpart of the pseudoscalar mesons and hence most strongly bound) and using the static approximation to the exchanged quark. Thereby the static approximation was shown to be sufficiently accurate in [11, 12]. Investigations of form factors and magnetic moments within an additive diquark–quark model show that the axial–vector diquarks are quite important even for the spin $1/2$ baryons [7]. The importance of axial–vector diquark correlations is also observed in the

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¹Let us also emphasise that in this approach it is not necessary that the diquarks are bound. The diquarks serve only as a convenient building block of the baryons. This is different in phenomenological diquark–quark descriptions of baryons [6, 7], where elementary diquarks are considered.

explicit numerical solutions of the Faddeev equation for the nucleon given in [13, 14]. In the present paper we will include the axial–vector diquarks and solve the Faddeev equation derived in [5] in the static approximation for both the $s = 1/2$ and $s = 3/2$ hyperons. For this purpose we will transform the relativistic Faddeev equation for s-wave bound quark-diquark states to a system of coupled Dirac and Rarita–Schwinger equations, respectively. This Poincaré invariant spin projection is much more involved than the spin algebra in the non-relativistic quark model or in its relativistic extension given e.g. in [15] and has so far not been carried out.

2. The hadronized NJL model

Let us briefly outline the relevant results of the hadronization of the color Nambu–Jona-Lasinio model (for details see [5]). The model is defined through the Lagrangian,

$$\mathcal{L} = \bar{q}(i\cancel{\partial} - m_0)q - \frac{g}{2} j_\mu^A j^{A\mu} . \quad (1)$$

Here, $j_\mu^A = \bar{q} \gamma^\lambda \gamma_A / 2 \gamma_\mu q$, denotes the color octet current, λ^A ($A = 1, \dots, 8$) are the $SU_C(3)$ Gell–Mann matrices and $m_0 = \text{diag}(m_{0u}, m_{0d}, m_{0s})$ the current quark mass matrix. By use of Fierz transformations the interaction can be transformed into a flavor–singlet quark–antiquark channel and a flavor–antitriplet quark–quark channel

$$-\frac{g}{2} j_\mu^A j^{A\mu} = \frac{g}{3} \left((\bar{q} \Lambda_\alpha q)(\bar{q} \Lambda^\alpha q) + (\bar{q} \Gamma_\alpha C q^T)(\bar{q}^T C \Gamma^\alpha q) \right) . \quad (2)$$

Here, the vertices in the meson and diquark channel are defined by

$$\Lambda^\alpha := 1_c \frac{\lambda^a}{2} O^a , \quad \Gamma^\alpha := \frac{i\epsilon^A}{\sqrt{2}} \frac{\lambda^a}{2} O^a , \quad \alpha \equiv (A, a, a) , \quad (3)$$

where ϵ^A_{BC} stands for the Levi–Civita tensor and λ^a ($a = 0, \dots, 8$) are the generators of $U_F(3)$ flavor group. O^a denote the Dirac matrices,

$$O^a \in \{i\gamma^5, 1, \frac{i\gamma^\mu}{\sqrt{2}}, \frac{i\gamma^\mu\gamma^5}{\sqrt{2}}\} , \quad (4)$$

and C is the charge conjugation matrix.

Though the Fierz transformation relates the coupling constants of the quark–antiquark and the quark–quark channels we will treat them as independent parameters in the actual calculations.

By means of functional integral techniques the quantum theory defined by the Lagrangian (1) has been converted into an effective hadron theory [5] where the baryons are constructed as bound diquark–quark states. Like in the bosonization of the NJL model [2] the meson fields appear as collective quark–antiquark states. In the vacuum the scalar meson field develops an expectation value, M_i ($i = u, d, s$), which signals the spontaneous breaking of chiral symmetry. This quantity, which represents the constituent quark mass, is defined by the Schwinger–Dyson equation

$$M_i = m_{0i} + \frac{g}{3} \int \frac{d^4 k}{(2\pi)^4} \text{tr}(G_i(k)) , \quad (5)$$

where $G_{ij}(k) = \delta_{ij}G_i(k) = \delta_{ij}(\not{k} - M_i)^{-1}$ is the constituent quark propagator. The physical mesons represent small amplitude fluctuations ϕ around the vacuum expectation value and the meson masses M_m are defined by the Bethe–Salpeter equation

$$\left(-\frac{3}{2g}\mathbf{g}_\alpha^\beta - \int \frac{d^4 k}{(2\pi)^4} \text{tr}[\Lambda_\alpha G(k + q/2)\Lambda^\beta G(k - q/2)] \right) \phi_\beta(q)|_{q^2=M_m^2} = 0 . \quad (6)$$

Here $\mathbf{g}^{\alpha\beta}$ is the metric tensor

$$\mathbf{g}^{\alpha\beta} = \delta^{AB}\delta^{ab}g^{ab} ; \quad g^{ab} = \begin{cases} \delta^{ab} & \text{for } O^a, O^b \in \{1, i\gamma^5\} \\ \delta^{ab}g^{\mu\nu} & \text{for } O^a, O^b \in \{i\gamma^\mu/\sqrt{2}, i\gamma^\mu\gamma^5/\sqrt{2}\} \end{cases} . \quad (7)$$

For the free baryon fields a relativistic Faddeev equation is obtained, which reads [5]

$$-\int \frac{d^4 q_2}{(2\pi)^4} L_\alpha^\gamma(p, q_1, q_2)\psi_\gamma(p, q_2) = 0 , \quad (8)$$

where,

$$L^{\alpha\gamma}(p, q_1, q_2) = iG\left(\frac{p}{2} + q_1\right)D^{\alpha\beta}\left(\frac{p}{2} - q_1\right)H^{\beta\gamma}(q_1, q_2) , \quad (9)$$

contains the two–body quark–quark correlations through the diquark propagator

$$(D^{-1})^{\alpha\beta}(q) = -\frac{3}{4g}\mathbf{g}^{\alpha\beta} - \frac{1}{2}\int \frac{d^4 k}{(2\pi)^4} \text{tr}[\Gamma^\alpha G(k + q/2)\Gamma^\beta G(k - q/2)] . \quad (10)$$

Furthermore H describes the exchange of a quark,

$$H^{\beta\gamma}(q_1, q_2) = \Gamma^\gamma G(-q_1 - q_2)\Gamma^\beta . \quad (11)$$

The relativistic Faddeev equation (8) has been rederived by more traditional means in [11, 12, 13] making explicit use of the separability of the NJL interaction.

Later on we will employ the static approximation to the exchanged quark [10],

$$G(q) = \frac{1}{\not{q} - M} \rightarrow -\frac{1}{M} , \quad (12)$$

which reduces the integral equation (8) to the algebraic equation

$$(\mathbf{g}_\alpha^\gamma - L_\alpha^\gamma)\phi_\gamma(p) = 0 , \quad (13)$$

where

$$L^{\alpha\gamma}(p) := \int \frac{d^4 q_1}{(2\pi)^4} L^{\alpha\gamma}(p, q_1) , \quad \phi^\gamma(p) := \int \frac{d^4 q}{(2\pi)^4} \psi^\gamma(p, q) . \quad (14)$$

Below we will include scalar ($a = 0$) and axial–vector ($a = 1$) diquarks, which are the counterparts of the pseudoscalar and the vector mesons and hence the most strongly

correlated diquark states of spin 0 and 1. The Faddeev equation (13) then acquires the form²

$$\begin{pmatrix} 1 - (L^{00}) & -(L^{01})^\nu \\ -(L^{10})_\mu & g_\mu^\nu - (L^{11})_\mu^\nu \end{pmatrix} \begin{pmatrix} \phi^0 \\ \phi_1^\nu \end{pmatrix} = 0 , \quad (15)$$

where ϕ^0 and ϕ_1^ν denote the diquark–quark amplitudes (see (14)) containing a scalar and an axial–vector diquark, respectively.

3. Spin projection

Let us discuss s-wave bound states of three relativistic constituent quarks from the group theoretical point of view. Relativistic constituent quarks, which are described by Dirac spinors belong to the representation $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ ³ of the homogeneous Lorentz group. The direct product of two quarks is reducible with respect to the decomposition

$$((\frac{1}{2}, 0) \oplus (0, \frac{1}{2})) \otimes ((\frac{1}{2}, 0) \oplus (0, \frac{1}{2})) = 2(0, 0) \oplus 2(\frac{1}{2}, \frac{1}{2}) \oplus ((1, 0) \oplus (0, 1)) .$$

Obviously, s-wave bound states of a quark and a (pseudo-)scalar diquark belonging to the $(0, 0)$ representation of homogeneous Lorentz group form baryons in the spin $\frac{1}{2}$ representation $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$. Bound states of a quark and an (axial-)vector diquark belonging to the $(\frac{1}{2}, \frac{1}{2})$ representation are represented by vector spinors. The vector spinor representation is reducible according to the homogeneous Lorentz group:

$$(\frac{1}{2}, \frac{1}{2}) \otimes ((\frac{1}{2}, 0) \oplus (0, \frac{1}{2})) = ((\frac{1}{2}, 0) \oplus (0, \frac{1}{2})) \oplus ((1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)) . \quad (16)$$

Since the irreducible Lorentz representation (j_1, j_2) contains the spins, $j = |j_1 - j_2|, \dots, j_1 + j_2$, the representation $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ contains both spin $\frac{1}{2}$ and spin $\frac{3}{2}$ components. To project the $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ representation onto good spin we split the vector spinor representation (16) into spin $\frac{1}{2}$ and spin $\frac{3}{2}$ representations of the Lorentz spin group. This is achieved by means of the following projectors,

$${}^{(\frac{1}{2}, 0)}\mathbb{P}_\mu^\nu = a_\mu a^\nu , \quad {}^{(1, \frac{1}{2} | \frac{1}{2})}\mathbb{P}_\mu^\nu = b_\mu b^\nu , \quad {}^{(1, \frac{1}{2} | \frac{3}{2})}\mathbb{P}_\mu^\nu = g_\mu^\nu - a_\mu a^\nu - b_\mu b^\nu , \quad (17)$$

where,

$$a_\mu = \frac{\gamma_\mu}{2} , \quad b_\mu = \frac{1}{\sqrt{3}} \left(\frac{\gamma_\mu}{2} - 2 \frac{\not{p} p_\mu}{p^2} \right) . \quad (18)$$

The spin $\frac{3}{2}$ projector obeys the Rarita–Schwinger constraints

$$\gamma^\mu {}^{(1, \frac{1}{2} | \frac{3}{2})}\mathbb{P}_\mu^\nu = 0 , \quad p^\mu {}^{(1, \frac{1}{2} | \frac{3}{2})}\mathbb{P}_\mu^\nu = 0 . \quad (19)$$

The QCD dynamics allows in principle also for tensor diquark correlations. The direct product representation of an antisymmetric tensor diquark and a quark field can be projected onto two spin $\frac{1}{2}$ and two spin $\frac{3}{2}$ representations. Thus, s-wave bound states of

²Flavor and color indices will usually be suppressed in the following.

³As it is well known the representations of the homogeneous Lorentz group can be characterized by the quantum numbers (j_1, j_2) of two SU(2) groups. The representations (j_1, j_2) contain representations of the Lorentz spin group with spin $j = |j_1 - j_2|, \dots, j_1 + j_2$, respectively.

three constituent quarks can give rise to eight spin $1/2$ and four spin $3/2$ baryon fields (see also [16]). We will restrict ourselves in the following to scalar and axial–vector diquark correlations. Then we have three spin $1/2$ and one spin $3/2$ baryon fields.

Classifying baryons according to the Lorentz spin group has the unpleasant feature that the projectors onto the vector spinor (and tensor spinor) subspaces do no commute with \not{p} . On the other hand the representations of the Poincaré group, which are characterized by the particle’s spin and mass, do commute with \not{p} . They can be formed by taking linear transformations of the Lorentz spin group. For the vector spinor representation (16) the corresponding projectors can be expressed as

$${}^l\mathbb{P}_\mu^\nu = l_\mu l^\nu , \quad {}^{tt}\mathbb{P}_\mu^\nu = t_\mu t^\nu , \quad {}^w\mathbb{P}_\mu^\nu = g_\mu^\nu - l_\mu l^\nu - t_\mu t^\nu = {}^{(1,1/2|3/2)}\mathbb{P}_\mu^\nu , \quad (20)$$

where,

$$l_\mu = \frac{1}{2}a_\mu - \frac{\sqrt{3}}{2}b_\mu = \frac{\not{p}p_\mu}{p^2} , \quad t_\mu = \frac{\sqrt{3}}{2}a_\mu + \frac{1}{2}b_\mu = \frac{1}{\sqrt{3}}(\gamma_\mu - \frac{\not{p}p_\mu}{p^2}) \quad (21)$$

and the momentum p_μ will be fixed later on by the equation of motion. The projectors ${}^l\mathbb{P}$, ${}^{tt}\mathbb{P}$, the trivial projector,

$${}^{ss}\mathbb{P} \equiv 1 = s s , \quad s = \gamma_5 , \quad (22)$$

and the operators,

$${}^{sl}\mathbb{P}^\nu = s l^\nu , \quad {}^{st}\mathbb{P}^\nu = s t^\nu , \quad {}^{lt}\mathbb{P}_\mu^\nu = l_\mu t^\nu , \quad (23)$$

$${}^{ls}\mathbb{P}_\mu = l_\mu s , \quad {}^{ts}\mathbb{P}_\mu = t_\mu s , \quad {}^{tl}\mathbb{P}_\mu^\nu = t_\mu l^\nu , \quad (24)$$

form a complete basis for all operators acting on spin $1/2$ s-wave baryons with scalar and axial–vector diquarks. In spin $3/2$ subspace ${}^w\mathbb{P}_\mu^\nu$ is the identity ⁴.

We now apply this spin projection formalism to the Faddeev equation (15). We define three Dirac spinor fields by the contractions,

$${}^s\phi := s\phi^0 , \quad {}^t\phi := t^\nu\phi_\nu^1 , \quad {}^l\phi := l^\nu\phi_\nu^1 , \quad (25)$$

and a spin $3/2$ Rarita–Schwinger vector spinor field by

$${}^w\phi_\mu := {}^w\mathbb{P}_\mu^\nu\phi_\nu^1 . \quad (26)$$

The kernel $L(p)$ defined by (9) and the first equation of (14) is a linear combination of the operators ${}^{s_1s_2}\mathbb{P}$, $s_i \in \{s, l, t\}$, and ${}^w\mathbb{P}_\mu^\nu$. Multiplying equation (15) from the left by s , l^μ , t^μ and ${}^w\mathbb{P}_\mu^\nu$ and using the decompositions $1 = ss$ and $g_\nu^{\bar{\nu}} = l_\nu l^{\bar{\nu}} + t_\nu t^{\bar{\nu}} + {}^w\mathbb{P}_\nu^{\bar{\nu}}$, the Faddeev equation reduces to a system of coupled Dirac equations for the spin $1/2$ baryon field,

$$\left({}^{s_1s_2}A(p^2) \not{p} - {}^{s_1s_2}B(p^2) \right) {}^{s_2}\phi(p) = 0 ; \quad s_i \in \{s, l, t\} , \quad (27)$$

⁴The operators ${}^l\mathbb{P}$, ${}^{tt}\mathbb{P}$, ${}^{lt}\mathbb{P}$, ${}^{tl}\mathbb{P}$ and ${}^w\mathbb{P}$ has been firstly derived in [18] from the Pauli–Lubanski vector in vector spinor representation.

and to a Rarita–Schwinger equation for the spin $3/2$ baryon field

$$\left({}^w A(p^2) \not{p} - {}^w B(p^2) \right) {}^w \phi_\nu(p) = 0 ; \quad \gamma^\mu {}^w \phi_\mu = 0 = p^\mu {}^w \phi_\mu . \quad (28)$$

Here, the momentum dependent quantities A and B following from equation (15) are defined by

$${}^{s_1 s_2} A \not{p} - {}^{s_1 s_2} B = \begin{cases} s_1 (1 - L^{00}) s_2 & \text{if } s_1 = s = s_2 \\ -s_1 (L^{01})^\nu s_{2\nu} & \text{if } s_1 = s, s_2 \in \{l, t\} \\ -s_1^\mu (L^{10})_\mu s_2 & \text{if } s_1 \in \{l, t\}, s_2 = s \\ s_1^\mu \left(g_\mu^\nu - (L^{11})_\mu^\nu \right) s_{2\nu} & \text{if } s_1, s_2 \in \{l, t\} \end{cases} \quad (29)$$

and

$$({}^w A \not{p} - {}^w B) {}^w \mathbb{P}_{\bar{\mu}}^{\bar{\nu}} = {}^w \mathbb{P}_{\bar{\mu}}^\mu \left(g_\mu^\nu - (L^{11})_\mu^\nu \right) {}^w \mathbb{P}_\nu^{\bar{\nu}} . \quad (30)$$

They involve the constituent quark propagator as well as the scalar and the axial–vector diquark propagator, see equation (14).

The axial–vector diquark propagator contains both a transverse and a longitudinal part,

$$D^{1,\mu\nu}(k) = \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) D^{1,trans}(k^2) + \frac{k^\mu k^\nu}{k^2} D^{1,long} . \quad (31)$$

Let us emphasize the importance of its longitudinal part. For Poincaré invariant theories the longitudinal part of a vector propagator is momentum independent and related to the transverse part by the relation, $D^{1,long} = D^{1,trans}(k^2 = 0)$ ⁵. The straightforward calculation of the quantities A and B defined above shows that it is the longitudinal degree of freedom, which ensures that the non–kinematic pole of the transverse axial–vector propagator at $k^2 = 0$ does not contribute to the Faddeev kernel. The pole only occurs in the form $(D^{1,trans}(k^2) - D^{1,long})f(k^2)/k^2$, where $f(k^2)$ is a regular expression for $k^2 \rightarrow 0$, and thus can be removed using the relation (31). The longitudinal part of the axial–vector diquark propagator cannot be discarded in the Faddeev equation because the diquark propagator is off energy shell. Neglecting it would give rise to ill–defined integrals.

Finally some remarks concerning the color and flavor degrees of freedom are in order. Color singlet and color octet baryons decouple since the quark exchange (11) has the color structur (see e.g. [10])

$$H_{LM}^{BC} \sim \frac{i\epsilon_{LK}^C}{\sqrt{2}} \frac{i\epsilon_{KM}^B}{\sqrt{2}} = - {}^{1_c} \mathbb{P}_{LM}^{BC} + \frac{1}{2} {}^{8_c} \mathbb{P}_{LM}^{BC} . \quad (32)$$

Transforming the Faddeev equation (8) into the usual representation of flavor singlet, ρ – and λ –type octet and decuplet baryons (see e.g. [19]) one easily shows that the $SU_F(3)$ multiplets do not decouple due to the explicit symmetry breaking by the strange quark mass⁶.

⁵For a detailed discussion of the Proca equation in the context of the Poincaré group we refer to [18].

⁶Note that a baryon state with a certain flavor content (e.g. uud or uds) need not belong to a specific $SU(3)$ –flavor representation.

4. Numerical results

The dynamical equations (27) and (28) have to be diagonalized in spin flavor space for the color singlet baryons. Due to Poincaré invariance and flavor conservation of the interaction (2), only components with the same spin and with the same flavor content couple. Furthermore, the Pauli principle forbids scalar flavor–sextet and axial–vector flavor–antitriplet diquarks⁷.

In the NJL model arising by Fierz transformation from the Lagrangian (1) the coupling constants in the (pseudo–)scalar meson channel, g_m^0 , in the (axial–)vector meson channel, g_m^1 , in the (pseudo–)scalar diquark channel, g_d^0 , and in the (axial–)vector diquark channel, g_d^1 , are related to each other. Since the local NJL interaction (2) is only a rather crude approximation to the low energy QCD dynamics, we prefer to chose different coupling constants in these channels. This is also tolerated by chiral symmetry. Then, the model has the following parameters: The current quark masses, $m_{0u} = m_{0d}$, m_{0s} , the coupling constants, g_m^0 and g_m^1 , g_d^0 , g_d^1 , and the ultraviolett cutoff parameter, Λ (see (6) and (10)). The static approximation (12) necessitates the introduction of a further cut–off for the momentum integral in the reduced Faddeev equation (see (13) and (14)). In principle, this cut–off is different from the low energy cut–off of the NJL model but to reduce the number of parameters we will choose both cut–offs to be identical. The numerical results are rather insensitive to the actual value of the cut–off in the Faddeev equation in a rather large range.

We perform the momentum integration in the first equation of (14) using a two pole approximation (see [10]) as well as an one pole approximation to the diquark propagator. The two pole approximation is in good agreement within the exact diquark propagator. Using the one pole approximation scalar and axial–vector diquarks are considered as Klein–Gordon and Proca particles, respectivly.

With a sharp $O(4)$ cut–off the longitudinal axial–vector diquark propagator is slightly momentum dependent and the relation $D^{1,long} = D^{1,trans}(k^2 = 0)$ is violated. In the present calculations we fix the longitudinal part of the axial–vector diquark propagator by means of this relation.

	p	Σ	Λ	Ξ	Δ	Σ^*	Λ^*	Ω
I	939	1159	1165	1379	1238	1440	1647	1859
II	939	1117	1162	1328	1238	1410	1589	1777
III	939	1094	1166	1303	1238	1411	1584	1768
exp	939	1116	1193	1318	1238	1385	1530	1675

Table 1: *The masses of the spin $1/2$ flavor–octet and the spin $3/2$ flavor–decuplet baryons: The results, indicated with (I), corresponds to the two pole approximation of the diquark propagator, whereas the results of the rows (II) and (III) are calculated by means of the one pole approximation. Constituent masses $M_u = 500\text{MeV}$ (I,II) and $M_u = 430\text{MeV}$ (III) of the nonstrange quark are used. The parameters are fixed as explained in the text.*

We fix the parameters, g_m^0 , Λ , $m_{0u} = m_{0d}$ and m_{0s} from the pion decay constant, $f_\pi = 93$ MeV, the pion mass, $m_\pi = 138$ MeV, and the kaon mass, $m_K = 496$ MeV, respectively.

⁷For a detailed discussion of the permutation symmetry of the relativistic three quark bound states we refer to [16, 17].

This leaves one free parameter, which we choose (via the gap equation (5)) to be the non-strange constituent quark mass, M_u . The coupling constants g_d^0 and g_d^1 are fixed by the proton and the Δ masses. The numerical results for the $1/2$ and $3/2$ baryon masses are presented in table 1. Using the two pole approximation to the diquark propagator, see row (I) of table 1, the ratio of the coupling constants in the scalar diquark and pseudoscalar meson sector, ($g_d^0/g_m^0 = 1.09$) deviates only slightly from unity, the value following from the Fierz transformation of the color octet NJL model (see (2)). The predicted baryon masses are in fair agreement with the experimental values except for the Ω mass, which is somewhat overestimated. This is mainly due to the large strange constituent mass ($M_s = 719\text{MeV}$ (I,II), $M_s = 665\text{MeV}$ (III)). We are forced to choose large constituent masses in order to have bound axial-vector diquarks, since our numerical routine breaks down for unbound diquarks. Note however that the diquarks need not bound on physical ground. Allowing also for unbound diquarks we expect a further improvement of our numerical results.

In this paper we have described flavor octet and decuplet baryons within the hadronized NJL model. Emphasis is put on a proper Poincaré invariant spin projection, which so far has not been performed in the present diquark-quark picture. Including scalar and axial-vector diquark correlations we have derived a system of coupled Dirac equations for the spin $1/2$ octet and of Rarita-Schwinger equations for the spin $3/2$ decuplet baryons. Our numerical calculations show, that the axial-vector diquark admixture to the spin $1/2$ baryons is indeed non-negligible. Finally, let us emphasize that the Poincaré invariant spin projection of the Faddeev equation presented here is not restricted to the NJL dynamic and can be straightforwardly extended to baryons with orbital excitations.

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Note added

One of the referees brought ref. [20] to our attention, which reports on the numerical calculation of hyperon masses within the hadronized NJL model. Although the numerical results are similar, our calculations differs in the following essential point: As can be seen from the text between equations (4) and (5) of ref. [20], C. Hanhart and S. Krewald neglect the longitudinal axial-vector diquark correlations. However, as discussed above, longitudinal axial diquarks are essential in order to obtain a well defined Faddeev kernel. They are included in the present work.

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